JAGIELLONIAN
UNIVERSITY
IN KRAKOW

Optimization and Control @ Jagiellonian University

Fifth German Polish Conference on Optimization Methods and Applications

Dobczyce, November 9 – 13, 2011

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Dear Participants,

Welcome to the 5th German Polish Conference on Optimization - Methods and Applications with special emphasis on Smooth and Nonsmooth Optimization, Variational Problems, Variational Inequalities, and Optimal Control (GPCO5), Dobczyce near Krakow, Poland. The present conference is a continuation of the German Polish Conference on Optimization series, which took place in Żagań (1999), Cottbus (2002), Będlewo (2005) and Moritzburg (2009).

We wish you a fruitful and pleasant stay in Dobczyce. We hope very much that during the GPCO5 you will be able to exchange your ideas, to present your research findings, and to share experiences with other researchers.

This book contains a list of the invited talks, remarks of general interest, the abstracts of the talks, and a list of the participants.

We would like to express our thanks to the Jagiellonian University in Krakow for the technical and financial support and to the Jałowcowa Góra resort in Dobczyce which hosts the meeting in its conference center and provides accommodation and full board by its facilities. Further we acknowledge the sponsorship kindly given by the Institute of Computer Science and the Faculty of Mathematics and Computer Science at the Jagiellonian University, the Committee of Mathematics of Polish Academy of Sciences, and by Deutsche Forschungsgemeinschaft (DFG). We thank the colleagues who have helped to prepare the conference, in particular dr hab. Leszek Gasiński, dr Krzysztof Bartosz, dr Piotr Kalita and dr Anna Ochal. Please do not hesitate to contact us in the case of questions or problems.

On behalf of the Steering Committee I wish you a successful meeting!

Stanislaw Migórski

Contents

General Remarks

- The registration takes place on Wednesday, November 9, in the ground floor of the main building of the Jałowcowa Góra resort and on the following days in the foyer of the lecture room.

- You will receive a copy of the program schedule in your welcome packet. All lectures will be given in the main building of the Jałowcowa Góra resort (Aula). Each plenary speaker has 45 minutes, all others have 20 minutes (excluding discussion). Any unexpected and last minute changes to the schedule will be posted on a flyer placed on the board near Aula.

- As a rule all participants are accommodated in the hotel places of the Jałowcowa Góra.

- Participants will find a free parking lot available near the main building of the resort.

- The registration fee includes conference documents, meals (breakfast, lunch, dinner), coffee and tea, social events, banquet and accommodation during the conference. The meals are served in the main building of the Jałowcowa Góra resort. Additional beverages are not covered by the conference. A choice of cold drinks will be available. Please pay individually what you consume according to the price list of the cafeteria.

- Refreshments will be provided during the morning and afternoon coffee breaks.

- Saturday afternoon, November 12, is reserved for individual discussions. Furthermore, an excursion is planned to the historic Salt Mine in Wieliczka (20 km from conference site). This is the only mining site in the world functioning continuously since the Middle Ages. The mine walk tour, under the supervision of a tour guide, consists of several chambers and an underground salt mining exhibition and takes about 3 hours.

- Saturday evening, November 12, the conference dinner will be held with an enriched buffet including a certain amount of beverages.

- Wi-Fi will be available in the foyer of the lecture hall. Password will be provided.

- It is planned to publish proceedings of GPCO5 in special issues of the international journal Schedae Informaticae. The manuscripts have to be submitted until April 30, 2012 to stanislaw.migorski@uj.edu.pl. All papers will be reviewed.

The address of the conference venue is following

Jałowcowa Góra
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Anna Ochal (Jagiellonian University in Krakow)

Invited Speakers

Francisco Facchinei (University of Rome "La Sapienza")
Talk: Monotone VI-constrained hemivariational inequalities: distributed algorithms and power control in ad-hoc networks

Ekkehard Köhler (Technische Universität Cottbus)
Talk: Dynamic network flows and applications in traffic optimization

Zdzisław Naniewicz (Cardinal Stefan Wyszyński University, Warsaw)
Talk: On economic equilibrium problem involving approximate convex functions

Anna Ochal (Jagiellonian University in Krakow)
Talk: Optimal control problems in nonsmooth contact mechanics

Oliver Stein (Karlsruher Institut für Technologie)
Talk: The T-stationarity concept in nonsmooth optimization

Boris Vexler (Technische Universität München)
Talk: A priori error analysis for space-time finite element discretizations of parabolic optimal control problems

A predictor-corrector algorithm for solving strongly nonlinear general noncovex variational inequalities

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Abstract

This paper presents a predictor-corrector algorithm for solving the strongly nonlinear general noncovex variational inequality, which is a new class of nonconvex variational inequalities involving three nonlinear operators. We establish the equivalence between the strongly nonlinear general nonconvex variational inequalities and the fixed point problem and show that the convergence of the predictor-corrector method only requires pseudomonotonicity, which is a weaker condition than monotonicity. Some special cases are also discussed.

Fractional Optimal Control Problem of An Anomalous Diffusion Process in Polar Coordinates

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Abstract

In this study, the analytical and numerical solutions of a fractional optimal control problem (FOCP) for an anomalous diffusion system defined in polar coordinates and axis-symmetric case are researched. The main aim of this work is foundation of the optimal control function which minimizes the performance index of the FOCP so that the performance index is defined as a functional of both state and control variables of the system. A FOCP and its numerical solution scheme was first proposed by Agrawal [1]. After that many papers have been published for FOCP of time fractional diffusion systems [2]. The anomalous diffusion system in terms of both Caputo time and fractional Laplacian space operators is considered in the present formulation. A spectral representation is used to characterized the space and control functions with the eigenfunctions [3]. Grünwald-Letnikov approximation is used to find the numerical solutions. The effectiveness of this numerical method is validated with some figures obtained by MATLAB. The mathematical description of the present problem is given as follows. Performance Index is

$$J(u) = \frac{1}{2}\int_0^1\int_0^1 \left[Ax^2(r,t) + Bu^2(r,t)\right] r\,dr\,dt,$$

where $x(r,t)$ and $u(r,t)$ are state and control functions; dynamics of the anomalous diffusion system is defined by

$${}_t^C D^\alpha x(r,t) = -\kappa_\beta (-\Delta)^{\frac{\beta}{2}} x(r,t) + u(r,t),$$

where ${}_t^C D^\alpha$ $(0 < \alpha \le 1)$ and $(-\Delta)^{\frac{\beta}{2}}$ $(1 < \beta \le 2)$ are the Caputo derivative and fractional Laplacian operators, respectively; $\kappa_\beta > 0$ represents the diffusion coefficient; initial and boundary conditions are

$$\begin{aligned} x(r,0) &= x_0(r), \\ x(1,t) &= 0 \quad (t > 0). \end{aligned}$$

Keywords: Fractional Laplacian, Caputo, Optimal Control, Anomalous Diffusion.

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Existence result for hemivariational inequality involving $p(x)$-Laplacian

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Abstract

In this paper we study the nonlinear elliptic problem with $p(x)$-Laplacian (hemivariational inequality). We prove the existence of a nontrivial solution. Our approach is based on critical point theory for locally Lipschitz functionals due to Chang [1].

Keywords: $p(x)$-Laplacian, Palais-Smale condition, mountain pass theorem, variable exponent Sobolev space.

1 Introduction

Let $\Omega \subseteq R^N$ be a bounded domain with a $\mathcal{C}^2$-boundary $\partial\Omega$ and $N > 2$. We study the following nonlinear elliptic differential inclusion with $p(x)$-Laplacian

$$\begin{cases} -\Delta_{p(x)} u - \lambda |u(x)|^{p(x)-2} u(x) \in \partial j(x, u(x)) & \text{a.e. on } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $p : \overline{\Omega} \to R$ is a continuous function satisfying $1 < p^- := \inf_{x\in\Omega} p(x) \leq p(x) \leq p^+ := \sup_{x\in\Omega} p(x) < N < \infty$, $p^+ \leq \hat{p}^* := \frac{Np^-}{N-p^-}$ and $j(x,t)$ is a function locally Lipschitz in the t-variable and measurable in x-variable. By $\partial j(x,t)$ we denote the subdifferential with respect to the t-variable in the sense of Clarke [2]. The operator $\Delta_{p(x)} u := \operatorname{div}(|\nabla u(x)|^{p(x)-2} \nabla u(x))$ is the so-called $p(x)$-Laplacian, which becomes p-Laplacian when $p(x) \equiv p$. Problems with $p(x)$-Laplacian are more complicated than with p-Laplacian, in particular, they are inhomogeneous and possesses "more nonlinearity".

2 Main Results

We prove the existence of a nontrivial solution for problem (1). Methods which we are used are based on the critical point theory for nonsmooth Lipschitz functionals of Chang [1]. By use of variational approach we can replaced the study of differential equations or differential inclusions by search of critical points of energy functionals associated with this equations or inclusions. Mountain Pass Theorem provide the critical points of functionals, and thus gives the solutions of problem (1).

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Numerical methods for partial differential inclusions of hyperbolic type

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Abstract

We deal with numerical approach for a solution of a hyperbolic differential inclusion. To this end we exploit two numerical schemes based on the Rothe method and Faedo-Galerkin one respectively. They both consist in constructing the sequences of solutions of approximating semi-discrete problems. We provide a convergence of the sequences to the exact solution of the given problem. We also study an error estimate for the both methods.

Keywords: Differential inclusion, Rothe method, Faedo-Galerkin method, contact problems, Clark subdifferential.

1 Introduction

There are lots of problems in contact mechanics which can be modelled by means of differential inclusions. There are also many situations in industry and everyday life, in which the contact phenomena appear permanently. In order to develop the knowledge about these processes one need to know the results of numerical approach and simulations, so the efforts directed to this goal are reasonable. In [1] the method of Rothe is studied for dynamic systems governed by parabolic and hyperbolic differential equations. Our goal is to adopt these results for systems involving multivalued, possibly nonmonotone term.

2 Main Results

In the most abstract setting we consider an evolution triple $V \subset H \subset V^*$, the spaces $\mathcal{V} = L^2(0,T;V), \mathcal{H} = L^2(0,T;H)$ and $\mathcal{V}^* = L^2(0,T;V^*)$, $\mathcal{W} = \{u \in \mathcal{V},\ u' \in \mathcal{V}^*\}$, with $T > 0$ and consider the following problem: find $u \in \mathcal{V}$, with $u' \in \mathcal{W}$ such that

$$\begin{gathered} u''(t) + Au'(t) + Bu(t) + \iota^* \partial J(\iota u') \ni f(t) \qquad \text{for a.e. } t \in (0,T) \\ u(0) = u_0, \;\; u'(0) = u_1, \end{gathered} \tag{1}$$

with $u_0, u_1 \in V$, where ∂J stands for a Clarke subdifferential of the locally Lipschitz functional $J : V \to \mathbb{R}$ and $\iota : V \to U$ is a compact operator and U is a given space. The inclusion (1) is considered in the space V^*. The operators $A, B : V \to V^*$ come from physical properties of considered system and are called the viscosity and elasticity operator respectively, the multivalued term ∂J comes from contact conditions, while right hand side f of (1) comes from the external forces. The idea of the Rothe method is to introduce a discretization of time interval $(0,T)$, replace the given derivatives by finite differences and finally solve the obtained stationary (eliptic) problem by means of spacial discretization and application Galerkin method. An alternative Faedo-Galerkin method is based on the inverse routine, first we apply the Galerkin discretization, which leads us to the system of ordinary differential inclusions, which are solved by means of time discretization. The goal of the presentation is to examine the effectiveness of the both methods.

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Methods for variational inequality problem over the fixed point set of a quasi-nonexpansive operator

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Abstract

Many convex optimization problems in a real Hilbert space $\mathcal{H}$ can be written as a variational inequality problem $VIP(\mathcal{F}, C)$ formulated as follows: Given a closed convex subset $C \subset \mathcal{H}$, find $\bar{u} \in C$ such that $\langle \mathcal{F}\bar{u}, z - \bar{u} \rangle \geq 0$ for all $z \in C$, where $\mathcal{F} : \mathcal{H} \to \mathcal{H}$ is a strongly monotone and Lipschitz continuous operator. We will consider a special case of $VIP(\mathcal{F}, C)$, where $C := \operatorname{Fix} T$ for a quasi-nonexpansive operator $T : \mathcal{H} \to \mathcal{H}$, i.e., an operator having a fixed point and satisfying $\|Tu - z\| \leq \|u - z\|$ for any $u \in \mathcal{H}$ and $z \in \operatorname{Fix} T$. A standard method for solving $VIP(\mathcal{F}, C)$ is the projected gradient method $u^{k+1} = P_C(u^k - \mu \mathcal{F} u^k)$. (see Theorem 46.C in [2]). Unfortunately, the method cannot be applied for solving $VIP\ (\mathcal{F}, \operatorname{Fix} T)$, because, in general, the metric projection $P_C u$, where $u \in \mathcal{H}$, cannot be evaluated explicitly. In this case one can apply the hybrid steepest descent method (HSDM), $u^{k+1} = Tu^k - \lambda_k \mathcal{F} T u^k$. For detais, see [1], where sufficient conditions for the convergence of the method are given. We will consider a generalized HSDM for solving $VIP(\mathcal{F}, \operatorname{Fix} T)$, $u^{k+1} = T_k u^k - \lambda_k \mathcal{F} T_k u^k$, where $T_k : \mathcal{H} \to \mathcal{H}$, $k \geq 0$, are quasi-nonexpansive operators. We will suppose that $\bigcap_{k=0}^{\infty} \operatorname{Fix} T_k \supseteq \operatorname{Fix} T$ and $\operatorname{Fix} T_k$ approximate $\operatorname{Fix} T$ in some sense. We will give sufficient conditions for the convergence of the generalized HSDM.

Keywords: variational inequality problem, hybrid steepest descent method, metric projection, subgradient projection.

References

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Application of pseudo-inverse maps in the optimal corrections of nonlinear systems

Grzegorz Ciesielski

This paper will present the new idea of the *pseudo-inverse maps* Φ^+ and Φ_k^+ (fig. 1.(a)-(b)) applied to the

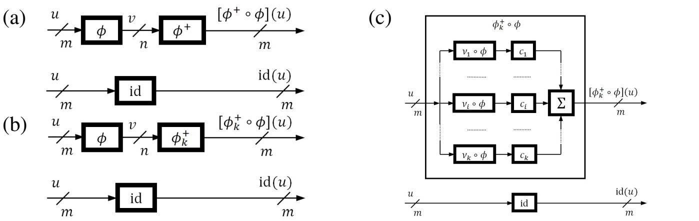

Fig. 1. Pseudo-inverse maps Φ^+ (a) and Φ_k^+ (b) of nonlinear system Φ as linear combination of elements of system ν_k (c)

opti- mal pre-, *feedforward* and *feedback corrections* by *correctors* γ_k of *nonlinear systems* Φ (fig. 2.(a)-(c) respectively).

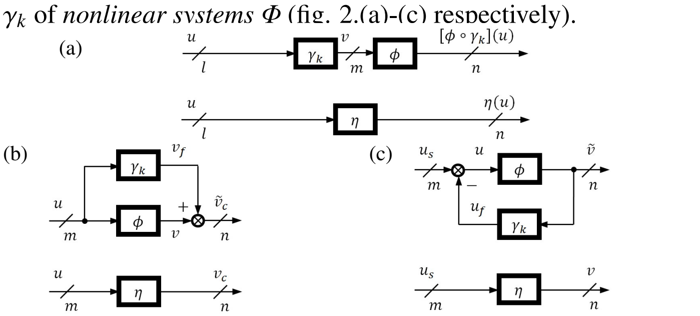

Fig. 2. Pre- (a), feedforward (b) and feedback (c) corrections by correctors γ_k of nonlinear systems Φ

This notion is the result of search for *optimal models* Φ_k of nonlinear systems Φ and its *optimal past-correctors* γ_k for the given *required system* η (fig. 3.(a)-(b) respectively)

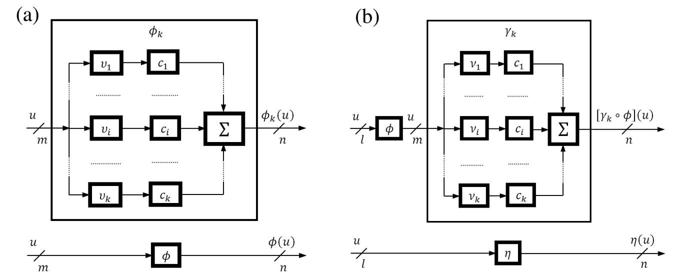

Fig. 3. Optimal model Φ_k of nonlinear systems Φ and its optimal past-corrector γ_k for the given required system η

from the perspective of the *Functional Theory of Nonlinear Systems* (*FTNS*) discovered and developed by author.

Considered systems Φ are multidimensional which means they have any finite number of *inputs* - m and *outputs* - n. The signals on inputs and outputs are real - $F = R$ or complex - $F = C$ valued, where F is a number field. Furthermore, the *input* and *output signals* are all on Abelian group G *time domain*, and they *sets* - $\mathcal{U}$ and $\mathcal{V}$ are equipped finally with the structure of the Hilbert spaces $\mathcal{U}_F$ and $\mathcal{U}_F$ (fig. 4).

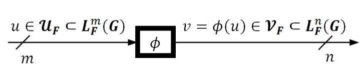

Fig. 4. System Φ as operator which maps the Hilbert space of inputs $\mathcal{U}_F$ in the Hilbert space of responses $\mathcal{V}_F$

Considered maps are *functions* for the *static sys- tems*, *multilinear convolutions*, defined by integral

$$\int_{R^k} u^T(t-\tau)\, v(\tau)\, d\tau ,$$

for the *linear time-invariant systems* and *nonlinear operators* for the nonlinear systems.

It is worth noting, that the nonlinear systems past-, pre-, feedforward and feedback corrections presented in this paper are reduced

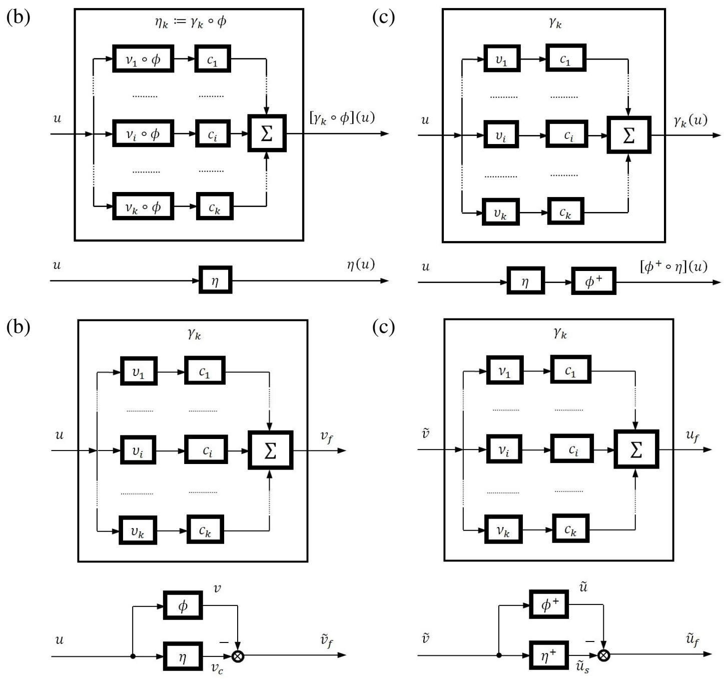

Fig. 5. Pre- (a), feedforward (b) and feedback (c) corrections by correctors γ_k of nonlinear systems Φ

to the modeling tasks of some systems (fig. 5.(a)-(d) respectively) which, in turn, can be reduced further to the *generalized least mean square* (*LMS*) *approximations*. For example, optimal model Φ_k of nonlinear systems Φ shown in fig. 3.(a) is

$$\Phi_k = c_{[k]} \upsilon_{[k]} ,$$

where

$$c_{[k]} := G^+(\upsilon_{[k]}) \langle \upsilon_{[k]}, u \rangle := G^+(\upsilon_{[k]}) (\upsilon_i, u)_1^k .$$

Some modeling and corrections results obtained by FTNS will be presented in the paper.

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Numerical solution for a variational-hemivariational inequality modeling an adhesive contact

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Abstract

We consider a mathematical model which describes the contact between a linearly elastic body and an obstacle. The process is static and frictionless. The normal contact is governed by two laws. The first one is Signorini law, representing the fact that there is no penetration between body and obstacle. The second one is a Winkler type law signifying that if there is no contact the bonding force is proportional to the displacement below a given bonding threshold and equal to zero above. The model leads to variational-hemivariational inequality. Our approach is inspired by Finite Element Method and is based on minimization of energy functional by means of Proximal Bundle Method (see [1]). The Bundle Methods have already been used to solve the elastic contact problems with adhesion force (see [2]), and contact problems with friction (see [3]). We present the numerical results for simple two-dimensional model problems. The simulations are accompanied by the numerical estimation of the order of the method, achieved by comparing solutions for increasingly dense meshes.

Keywords: linearly elastic body, Winkler law, unilateral contact, adhesion, variational-hemivariational inequality, proximal bundle method

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Optimality conditions for bilevel programming problems

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Abstract

Bilevel programming problems are hierarchical optimization problems where the feasible region is (in part) restricted to the graph of the solution set mapping of a second parametric optimization problem. To solve them and to derive optimality conditions for these problems this parametric optimization problem needs to be replaced with its (necessary) optimality conditions. This results in a (one-level) optimization problem. In the talk different approaches to transform the bilevel programming will be suggested and the relations between the original bilevel problem and the one replacing it will be investigated. The resulting (necessary) optimality conditions will be formulated.

Level control in projection methods for convex minimization

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Abstract

We study a projection method with level control for convex minimization problems. The level estimates the minimal value of the objective function (convex, not necessarily differentiable) and is updated in each iteration (see [4] and [1]). We introduce a changeable level parameter to level control and analyse the convergence of this method (see [2]). The idea of level control is similar to proposed for subgradient method of Polyak in [3]. We also present numerical results for some test problems.

Keywords: projection method, level control, convex nondifferentiable minimization.

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Optimization problems in stability of switched systems

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Abstract

Switched systems are continuous-time systems with discrete switching events from a certain class. The motivation for studying switched systems comes partly from the fact that switched systems and switched multi-controller systems have numerous applications in the control of mechanical systems, process control, automotive industry, power systems, aircraft and traffic control, intelligent control and many other fields.

For switched systems, it is natural to consider the existence of a common diagonal, common quadratic, common polyhedral and common polynomial Lyapunov functions. Each of these problems is related to the corresponding optimization problem.

In this report, we review optimization problems arising in the theory of stability of switched system. In the second part, we consider the solution of the problem of common quadratic Lyapunov function by convex optimization methods.

Keywords: Switched systems, Lyapunov function, Gradient algorithms.

Monotone VI-constrained hemivariational inequalities: distributed algorithms and power control in ad-hoc networks

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Abstract

We consider centralized and distributed algorithms for the numerical solution of a hemivariational inequality (HVI) where the feasible set is given by the intersection of a closed convex set with the solution set of a lower-level monotone variational inequality (VI). The algorithms consist of a main loop wherein a sequence of one-level, strongly monotone HVIs are solved that involve the penalization of the non-VI constraint and a combination of proximal and Tikhonov regularization to handle the lower-level VI constraint. The methods developed are then used to successfully solve a new power control problem in ad-hoc networks.

New penalty-type methods for bilevel optimization

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Abstract

We present new approaches for bilevel optimization, derived from an optimality condition for the lower level problem that leads naturally to a nonsmooth penalty function. The nonsmoothness of the penalty stems from a projection operator that project directions of descent for the lower level function onto the set of feasible points of the lower level problem. Preliminary numerical results on bilevel problems occuring in electricity markets show the efficacy of the approach. Further, we consider possible extensions to the multilevel case and to the case where the lower level problem is a multiobjective problem. Finally, we highlightconnections of the new method with the multiobjective approach to bilevel optimization as presented by Fliege & Vicente (2004).
Keywords: bilevel, multiobjective, penalty, Pareto, multilevel, projection, optimality conditions

Optimization of measurement deviations

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Abstract

The monitoring of production processes requires to compare the obtained workpieces with its design. For this purpose the workpiece is scanned and described by measurings [3]. From these measuring ideal models have to be derived and those finally are compared with the design parameters. The measurements yield a huge number of data points. The mathematical problem is to find parameters of geometrical objects like circles, planes and cylinders that fit the data points in an appropriate sense - Gauss fitting, minimum circumscribed, maximum inscribed or minimum zone fitting [4]. The Gauss fitting is to minimize the quadratical distance from the data points to the geometrical object. This is a nonlinear optimization problem. The minimum zone fitting is to minimize the maximum of the absolute distance between the data points and the geometrical object. This optimization problem which is just a classical minimax problem. The maximum inscribed and the minimum circumcribed can be transform into a minimax-problem too. The challenging is to develop fast optimization algorithms that may perform the given tasks in real time. In our talk we propose several optimization techniques [1],[2] for this specific problem and study its convergence behavior. Finally numerical tests are given.

Keywords: Tschebyscheff, geometrical form elements, measuring, MiniMax problem.

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On the existence of three solutions for quasilinear elliptic problem

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Abstract

We prove the existence of at least three solutions for a quasilinear elliptic problem using the so called three critical points theorem due to Ricceri.

Keywords: minimax inequality, p-Laplacian, three critical points theorem.

1 Introduction

We prove that the problem

$$\begin{cases} -\Delta_p u \;=\; \lambda(f(u)+\mu g(u)) \text{ in } \Omega, \\ u|_{\partial\Omega} = 0 \end{cases} \tag{1.1}$$

has at least three weak solutions in Sobolev space $W_0^{1,p}(\Omega)$. Here Δ_p stands for the p-Laplacian defined by $\Delta_p := \operatorname{div}\big(|\nabla \cdot|^{p-2}\nabla \cdot\,\big)$ with $p \in (1, +\infty)$.

2 Main Results

Theorem 2.1. *Let $\Omega \subset \mathbb{R}^N$ be an open and bounded set with smooth boundary and f, $g\colon \mathbb{R} \to \mathbb{R}$ continuous functions. Put*

$$F(\xi) \;:=\; \int_0^\xi f(t)\,dt, \qquad G(\xi) \;:=\; \int_0^\xi g(t)\,dt.$$

Assume that $\sup_{\xi\in\mathbb{R}} F(\xi) > 0$ and that there exist four positive constants a, q, s, γ with $s \in [1,p)$, $\gamma \in \big(p,p^*\big)$ and $q \in \big(0,p^*-1\big)$, such that

$$\begin{aligned} \max\big\{|f(\xi)|,|g(\xi)|\big\} \;&\leq\; a\big(1+|\xi|^q\big), \\ \max\{F(\xi),|G(\xi)|\} \;&\leq\; a\big(1+|\xi|^s\big) \end{aligned}$$

for every $\xi \in \mathbb{R}$ and

$$\limsup_{\xi\to 0} \frac{F(\xi)}{|\xi|^\gamma} \;<\; +\infty.$$

Then, there exists $\delta > 0$ such that, for each $\mu \in [-\delta,\delta]$, there exist $\rho > 0$ and an open interval $\Lambda \subset [0,+\infty)$ such that, for each $\lambda \in \Lambda$, the problem (1.1) has at least three weak solutions in $W_0^{1,p}(\Omega)$ whose norms are less than ρ.

In the proof of Theorem 2.1 we use the generalization of Proposition B.10 of Rabinowitz [1] and three critical points theorem [2, Theorem 1].

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A unified theory of first and second order optimality conditions for vector optimization problems

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In the talk we present a unified approach to deriving first and second order local optimality conditions, both necessary and sufficient, for solutions of vector optimization problems with non-solid positive cone.

Let X and Y be Banach spaces over the real field $\mathbb{R}$. The space Y is supposed to be ordered by a strict partial order $\prec$ such that for any $y_1, y_2 \in Y$ one has $y_1 \prec y_2$ if and only if $y_2 - y_1 \in P$, where $P \subset Y$ is an asymmetric convex cone with an empty interior.

The talk deals with a vector optimization problem

$$(VOP) \qquad \prec\text{–minimize}\, F(x) \text{ subject to } x \in Q,$$

where $F : X \to Y$ is a mapping from X into Y, Q is a subset of X.

A point $x^0 \in Q$ is called *a local $\prec$–minimizer* for (VOP) if there exists a neighborhood $N(x^0)$ of x^0 such that $F(x) \not\prec F(x^0)$ for all $x \in Q \cap N(x^0)$.

Firstly we reduce the vector optimization problem (VOP) to the variational system consisting of the scalar inequality and the operator equality. To do it we assume that the vector subspace $P - P$ coinciding with the linear hull of P is topologically complemented in Y and $\mathrm{ri}P \neq \varnothing$, where $\mathrm{ri}P$ is the relative interior of P. These assumptions allows us to choose a projection operator $\pi : Y \to Y$ with $\ker\pi = P - P$ and a sublinear function $\sigma : Y \to \mathbb{R}$ such that

$$\mathrm{ri}\, P_\prec = \{y \in Y \mid \sigma_\prec(-y) < 0,\, \pi_\prec(y) = 0\}, \quad \mathrm{cl}\, P_\prec = \{y \in Y \mid \sigma_\prec(-y) \le 0,\, \pi_\prec(y) = 0\}$$

and to prove the following reduction theorem.

a) If a point $x^0 \in Q$ is a local $\prec$–minimizer of the mapping $F : X \to Y$ over the set $Q \subset X$ then there exists a neighborhood $\mathcal{N}(x^0)$ of the point x^0 such that

$$\sigma(F(x) - F(x^0)) \ge 0 \textit{ for all } x \in Q \cap \mathcal{N}(x^0) \textit{ such that } \pi_\prec(F(x) - F(x^0)) = 0. \qquad (1)$$

b) If there exists a neighborhood $\mathcal{N}(x^0)$ of a point $x^0 \in Q$ such that

$$\sigma(F(x) - F(x^0)) > 0 \textit{ for all } x \in Q \cap \mathcal{N}(x^0),\ x \neq x^0, \textit{ such that } \pi_\prec(F(x) - F(x^0)) = 0, \quad (2)$$

then the point x^0 is a local $\prec$–minimizer of the mapping $F : X \to Y$ over the set $Q \subset X$.

Then we analyze (1) and (2) with variational methods and derive both necessary and sufficient $\prec$–minimality conditions for local $\prec$–minimizers of (VOP).

Analyzing (1) and (2) we assume that the objective mapping F is differentiable in one or another sense, in particular, we admit that F is twice parabolic directionally differentiable. As local approximations for the feasible set Q and the set $E(x^0)$ we use first- and second-order tangent vectors. In the case when the objective mapping F is twice Frechet differentiable and the feasible set Q is the whole space X the $\prec$–minimality conditions are presented in the prime and dual forms. The first order dual necessary $\prec$–minimality condition has the form of the Lagrange multipliers rule while the second order dual necessary $\prec$–minimality condition asserts that the maximum of the family quadratic forms on the cone of critical vectors is nonnegative. Note that necessary $\prec$–minimality conditions are obtained under the additional assumption that a local $\prec$–minimizer satisfies the regularity condition.

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Optimal control of a drying process with constraints to avoid cracks

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Abstract

The paper deals with the numerical treatment of the optimal control of drying of materials which may lead to cracks. The drying process is controlled by temperature, velocity and humidity of the surrounding air. The state equations define the humidity and temperature distribution within a simplified wood specimen for given controls. The elasticity equation describes the internal stresses under humidity and temperature changes. To avoid cracks these internal stresses have to be limited. The related constraints are treated by smoothed exact barrier-penalty techniques. The objective functional of the optimal control problem is of tracking type. Further it contains a quadratic regularization by an energy term for the control variables (surrounding air) and barrier-penalty terms.

The necessary optimality conditions of the auxiliary problem form a coupled system of nonlinear equations in appropriate function spaces. This optimality system is given by the state equations and the related adjoint equations, but also by an approximate projection onto the admissible set of controls by means of barrier-penalty terms. This system is discretized by finite elements and treated iteratively for given controls. The optimal control itself is performed by quasi-Newton techniques. .

Keywords: Optimal control, coupled parabolic system, penalties, discretization.

1 Introduction

The considered models lead to systems of two parabolic partial differential equations in three spatial variables. This pde system incorporates also phase changes which e.g. occur in the coupling terms of the system. Diffusion coefficients depend in a complex way on the state of the system and these dependencies have to be estimated by experiment. The same holds for the parameters of the evolution equations which describe phase changes. In our model the relevant state variables are temperature and moisture content of the material. This leads to an optimal control problem with a coupled system of time dependent initial-boundary value problems as state equations (cf. [3]). The numerical solution of the direct problem requires discretization in time and space. Time discretization is done by the implicit Euler method. The resulting elliptic system is discretized by the conforming Finite Element method on a triangular grid with linear elements. The drying process is controlled via the boundary values, namely temperature, moisture and velocity of the surrounding air. The objective of the process is to reduce the moisture of wood up to a prescribed degree under minimal energy. In order to avoid cracks (see [2]), restrictions on internal tensions are treated by a smoothed exact penalty method (see [1]). An efficient evaluation of gradients via the adjoint system for the time-discretized model is applied.

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Second-order optimality conditions for control problems with pure state constraints

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Abstract

We propose new second-order optimality conditions for optimal control problems with pure state constraints.

Keywords: Optimal control, Second-order necessary optimality conditions.

1 Introduction

We consider the following optimal control problem of the Bolza form:

$$\text{Minimize} \int_0^1 l(t, x(t), u(t))dt \qquad (P)$$

over measurable controls $u(\cdot)$ and solutions x of the control system

$$\begin{cases} \dot{x}(t) = f(t, x(t), u(t)) & u(t) \in U(t) \quad a.e. \\ x(0) = x_0 \\ x(t) \in K, \quad \forall t \in [0,1] \end{cases}$$

where the maps $f\colon [0,1] \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$, $l\colon [0,1] \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$, the set-valued map $U\colon [0,1] \rightsquigarrow \mathbb{R}^m$, the subset $K \subset \mathbb{R}^n$ and the initial state $x_0 \in \mathbb{R}^n$ are given.

Second-order optimality conditions for optimal control problems have been studied for almost half a century but it remains a challenge to formulate them in a very general context. In some recent works (see [1, 4]), second-order necessary optimality conditions for problems with pure state constraints were obtained by using an abstract infinite dimensional optimization problem. In the absence of pure state constraints, second-order necessary optimality conditions can be obtained by using a variational approach (see for instance [2, 3], where piecewise continuous controls were investigated). Inspired by these variational techniques, we use a variational approach to deduce new second-order necessary optimality conditions for the problem (P).

2 Main Results

Our second-order necessary conditions contain, in addition to the usual second-order derivative of the Hamiltonian, extra terms involving second-order tangents to the set of admissible trajectory-control pairs at the extremal process under consideration. Furthermore, our approach allows a direct proof in which we use a new second-order variational equation. Also, it allows to separate the proofs of the first- and second-order necessary conditions. In particular, our result applies to any first-order necessary optimality conditions in the form of the constrained maximum principle. Another important advantage is that the presence of pure state constraints does not lead to any restrictions on the control constraints, i.e. the result is applicable for any measurable set-valued map U. Finally, we do not assume any regularity of the optimal control other than measurability and also the dynamics of the control system are allowed to be merely measurable in the time variable t.

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A Gradient Calculus and a Numerical Scheme for a Nonlinear Inverse Heat Conduction Problem

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Abstract

We consider a nonconvex criterion of a two frontiers problem for a composite 1D rod in a heat conduction framework. A gradient algorithm together with a new numerical scheme are used to minimized it and find the two frontiers.

Keywords: heat equation, inverse problem, gradient, adjoint state, Haar wavelets.

1 Issues

A 1D rod of fixed length $b = b_1 + b_2 + b_3$ and composed of three materials is heated at each boundary with some given periodic fluxes of modules v_1, v_2 generating a complex thermal wave. We are looking for the two frontiers $b_1 = s_1 b$, $b_1 + b_2 = s_2 b$ with $s_1, s_2 \in [0,1], s_2 > s_1$. The complex thermal wave allows us to work with an ODE's system in space ξ simpler than the original PDE's system in time-space t, ξ. A normalization in space $\xi = b_i\zeta, \zeta \in [0,1]$ leads to :

$$j\omega C_i b_i^2 x_i(\zeta) - \lambda_i x_i"(\zeta) = 0, \quad i = 1,2,3, \tag{1}$$

$$\begin{cases} b_1 h_1 x_1(0) - \lambda_1 x_1'(0) = b_1 v_1 \\ x_1(1) = x_2(0), \quad \lambda_1 b_2 x_1'(1) = \lambda_2 b_1 x_2'(0) \\ x_2(1) = x_3(0), \quad \lambda_2 b_3 x_2'(1) = \lambda_3 b_2 x_3'(0) \\ b_3 h_2 x_3(1) + \lambda_3 x_3'(1) = b_3 v_2 \end{cases} \tag{2}$$

With two measures $\hat{x}_1, \hat{x}_2$ of the temperature at each end of the rod, we search the unknowns s_1^*, s_2^* as minimizers of the nonconvex criterion

$$J(s_1, s_2) = \frac{1}{2}\left(|x_1(0) - \hat{x}_1|^2 + |x_3(1) - \hat{x}_2|^2\right).$$

To that end, we calculate the gradient of J. The linear system (1), (2) of constraints has a trivial solution with exponentials. However the underlined shooting method is unfeasible for that type type of boundary value problem because it leads to an ill-conditionned problem. So we propose a new numerical scheme based on a Haar wavelets decomposition.

2 Main Results

With $\psi_i(\zeta), i = 1,2,3$ the solution of an adjoint system, $\frac{\partial J}{\partial s_1}$ for instance is the real part of

$$b\left(2\int_0^1 \left(\overline{\psi_2(\zeta)} b_2 (f_2(\zeta) - j\omega C_2 x_2(\zeta)) - \overline{\psi_1(\zeta)} b_1 (f_1(\zeta) - j\omega C_1 x_1(\zeta))\right) d\zeta \right.$$
$$\left. + \left((v_1 - h_1 x_1(0))\overline{\psi_1(0)} - \frac{\lambda_2 x_2'(0) + \lambda_1 x_1'(0)}{b_2}\overline{\psi_1(1)}\right) + \frac{\lambda_3}{b_2} x_3'(0)\overline{\psi_3(0)}\right)$$

The Haar wavelet method is an adaptation of [1] to the multidimensional case.

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Convergence of a variable time step θ-scheme for a parabolic hemivariational inequality

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Abstract

Let $V \subset H \subset V^*$ be an evolution triple where V is a reflexive Banach space, H is a Hilbert space and the embeddings are continuous, dense and compact. Let furthermore U be a reflexive Banach space, and let $\iota : V \to U$ be a linear, continuous and compact mapping such that $\iota(V)$ is dense in U. Let furthermore $T > 0$, $u_0 \in H$ and $f \in L^2(0, T; V^*)$. We consider the weak solutions to the parabolic partial differential inclusion of type

$$u'(t) + Au(t) + \iota^* \partial J(\iota u(t)) \ni f(t),$$

where the multivalued term is given by the subdifferential in the sense of Clarke of a locally Lipschitz functional J and the elliptic nonlinear operator A is pseudomonotone, coercive and satisfies the appropriate growth condition. The initial condition is given by $u(0) = u_0$. We consider the following implicit time stepping algorithm

$$\frac{u^k - u^{k-1}}{\tau_k} + Au^{k-1+\theta} + \iota^* \partial J(\iota u^{k-1+\theta}) \ni f^{k-1+\theta},$$
$$u^0 = u_0,$$

where $u^{k-1+\theta} = (1-\theta)u^{k-1} + \theta u^k$ and $f^{k-1+\theta} = (1-\theta)f^{k-1} + \theta f^k$, and the piecewise constant function $f_\tau(t) = f^k$ for $t \in (\tau_{k-1}, \tau_k]$ approximates the source term f. In each time step the elliptic problem is solved for the consecutive values u^k, starting from the initial condition. We give the assumptions under which the piecewise constant and piecewise linear interpolants built of the values $\{u^0, u^1, \ldots, u^K\}$ (where $\tau_K = T$) converge in appropriate sense to the solution of the original problem over the sequence of time meshes such that $\max_{k=1,\ldots,K} \tau_k \to 0$.
The fully implicit case ($\theta = 1$) has been considered in [2]. The case with arbitrary $\theta \in (0, 1)$, variable time step length and the monotone and hemicontinuous operator A has been considered for the equation (the case without the multivalued term) in [1]. Presented result extends the results of both mentioned articles.
Keywords: differential inclusion, Clarke subdifferential, time approximation, θ scheme, variable time step, convergence.

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Modifications and decompositions in sequences of gradients

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Abstract

Let $\Omega \subseteq \mathbf{R}^n$ be a bounded open domain, and let $\{u_k\}_{k\in\mathbf{N}}$ be a bounded sequence in Sobolew space where $1 \leq p < \infty$. Further, let $\mathcal{R}$ be a given complete separable ring of bounded continuous functions on $\mathbf{R}^{m\times n}$. Assume that: 1) u_k converges weakly to $u \in W^{1,p}(\Omega, \mathbf{R}^m)$, 2) for every $f \in \mathcal{R}$ the sequence of compositions $\{f(\nabla u_k)(1+|\nabla u_k|^p)dx\}_{k\in\mathbf{N}}$ embedded into the space of measures on $\bar{\Omega}$ converges weakly $*$ to some measure μ_f.
We discuss the possibility to modify the sequence $\{u_k\}_{k\in\mathbf{N}}$ in such a way that the new sequence $\{w_k\}_{k\in\mathbf{N}}$ is still bounded in $W^{1,p}(\Omega, \mathbf{R}^m)$, satisfies property 1) and 2) with the same function u and measures μ_f, and additionally "$w_k = u$" on F, where $F \subseteq \bar{\Omega}$ is a given closed set .

As one of the applications we deal with the variant of Decomposition Lemma due to Kinderlehrer and Pedregal asserting that an arbitrary bounded sequence of gradients of Sobolev mappings $\{\nabla u_k\} \subseteq L^p(\Omega, \mathbf{R}^{m\times n})$, where $p > 1$, can be decomposed into a sum of two sequences of gradients of Sobolev mappings: $\{\nabla z_k\}$ and $\{\nabla w_k\}$, where $\{\nabla z_k\}$ is equintegrable and carries the same oscillations, while $\{\nabla w_k\}$ carries the same concentrations as $\{\nabla u_k\}$. We additionally impose the general trace condition "$u_k = u$" on F, where F is given closed subset of $\bar{\Omega}$. Applying modification results we show that under this assumption the sequence $\{z_k\}$ in decomposition can be chosen to satisfy also the trace condition $z_k = u$ a.e. on F.

Both results are applied to nonconvex variational problems, in particular to regularity results for sequences minimizing functionals. As the main tool we use DiPerna Majda measures.

Keywords: sequences of gradients, DiPerna Majda measures, concentrations, oscillations.

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Recent advances in chance-constrained optimization - an overview

Michael Klöppel, Armin Hoffmann, Abebe Geletu, Pu Li

Chance-constrained optimization (CCOPT) has evolved in the last years with the development of new solution approaches, including, among others, the use of polynomial chaos, sparse grid integration methods and approximation schemes proposed by Nemirovski/Shapiro. These new methods allow for faster computation and the solution of more general (e.g. non-monotonic) problems. In this talk we consider CCOPT problems with single chance-constraints given by

$$\begin{aligned} \min_{u \in \mathcal{U}} \quad & E\big(f(u, y, \xi)\big) \\ s.t. \quad & g(u, y, \xi) = 0 \\ & P(y_{i_{min}} \leq y_i \leq y_{i_{max}}) \geq \alpha_i, \quad i \in I \\ & \xi \in \mathcal{X}. \end{aligned}$$

The basic ideas of the new approaches will be introduced, followed by a comparison regarding suitability and efficiency. Examples from engineering are taken to illustrate CCOPT.

Dynamic network flows and applications in traffic optimization

Ekkehard Köhler

Abstract

Network flow theory is a standard area of combinatorial optimization. Various versions of network flows have been defined, many efficient algorithms exist for optimizing flows and a variety of models, based on network flows have proved to be helpful in many practical applications. Yet, there are cases when standard static flows are not sufficient to capture the properties of a problem. In particular when the time-dependent behaviour of flow particles have to be modeled static flows are not complex enough. However, already in the early days of network flows, Ford and Fulkerson defined *dynamic network flows*, often also called *flows over time*. In these flows the flow particles move through the network over time.

In this talk we will focus on different algorithms for dynamic network flows for applications in traffic optimization. We will start by reviewing some standard results on static paths and flows and extend them to time-dependent flow models. In particular we will look at the traffic assignment problem, some results on quickest flows and a new model on optimizing the synchronization of traffic signals.

Optimal investment problem in the capital accumulation model with age structure

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Abstract

The capital accumulation model, where the capital goods may differ in age (*vintage*), is described by system (compare [1], [2]):

$$\begin{cases} u_t + a(t,x)u_x = -\mu(t,x)u + \alpha(t,x) & (t,x) \in (0,T] \times R \\ u(0,x) = f(x) & x \in R \end{cases} \tag{1}$$

Coefficients $\mu(t,x)$ and $\alpha(t,x)$ denotes depretiation factor and total investment at time t in capital goods of age x, respectively. We assume those functions to be continuous. $a(t,x)$ is a diffusion factor, which represents the aging process in technologies. The damage of machine or replacement the old technology by new one may be interpreted as a discontinuity in productivity, therefore it's quite natural to consider $a(t,x)$ as discontinuous function. Such problem posses unique viscosity solution.

We will consider the following control problem. Find the optimal investment strategy to maximize profit functional

$$J(u,\alpha) = \int_0^T \int_0^\infty e^{-\rho(x)t} \left[\kappa(t,x)u(t,x) - q(t,x)\alpha(t,x) - \beta(t,x)\alpha^2(t,x)\right] dxdt$$

where $\rho(x)$ is a discount factor, $\kappa(t,x)$ – a productivity factor, $q(t,x)$ – unit investment cost and $\beta(t,x)$ – unit adjustment cost. Maximization is over the set of admissible pairs (u,α), where α denotes investment and u is the viscosity solution of (1) with α set. It will be shown that under suitable assumptions the optimal investment strategy exists and is unique.

Keywords: optimal control, capital accumulation model, vintage structure.

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MSE approach to optimal control

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Abstract

Our aim is to present principles of the MSE approach to computational optimal control, developed by the authors for some time [1-4]. The MSE (*Monotone Structural Evolution*) is a direct numerical method, sequential in its basic version, using variable decision spaces in the induced NLP problems. Changes of decision space, or *structural changes*, are separated by periods of gradient optimization in a constant space. The MSE may be applied to ODE, hybrid, DAE and DDE systems. Control and some state constraints are directly dealt with, other constraints may be treated by penalty techniques.

Every control in the MSE is defined by a finite sequence of *control procedures* and a finite number of real parameters. This sequence is altered in structural changes, by removing and/or adding procedures from a predetermined stock. The rules for structural changes are so devised that every stationary point of the algorithm satisfies the Pontriagin maximum principle (MP). During optimization, the cost decreases monotonously, as the current control is not immediately affected by a structural change. If the stock of control procedures is sufficiently rich, it may be expected that the algorithm will produce an MP-consistent approximation of extremal solution [4], that is, lying in a finite dimensional space which also contains the exact extremal solution (with some exceptions, e.g., problems with Fuller's effect). One of practically important consequences of the general MSE concept is that typically, the final approximation of optimal control is fully represented by a relatively small number of parameters. The essential role of adjoint variables is characteristic of the method. Besides gradient computations, they serve for assessment of *efficiency* of structural changes and for triggering the most efficient of them, subject to the *rule of minimum positive efficiency* which prevents the algorithm from convergence to chattering modes.

Numerical aspects together with illustrative examples are discussed in the accompanying contribution [5].

Keywords: Optimal control, Dynamic optimizations, Pontriagin maximum principle.

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Order subdifferentials of monotone functionals

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Abstract

Our aim is to present further results concerning order subdifferentials of monotone functionals $f : X \longrightarrow \overline{R}$ over locally convex solid *Riesz* spaces X with applications to optimization. Among others $\vee-$ and $\wedge-$ subdifferentials $\partial^{\vee} f(x)$ and $\partial^{\wedge} f(x)$, respectively, are considered. The order subdifferentials are refered to monotone functionals which are convex or concave, respectively, to obtain further properties.

Keywords: subdifferential, directional derivative, monotone functional, smoothness, uniform smoothness, optimization, Riesz space

1 Introduction

A functional f is said to be monotone if $f(x) = f(|x|)$ and $f(y) \leq f(x)$ whenever $0 \leq y \leq x$. Let ([1]) $\partial^{\vee} f(x) \equiv \{x^* - y^* : \langle x, y^* \rangle = f(x) + f^*(x^*),\ 0 \leq y^* \leq x^*\}$ and let $\partial^{\wedge} f(x) \equiv \{x^* - y^* : \langle x, y^* \rangle = f(0) + f^+(x^*),\ 0 \leq y^* \leq x^*\}$ be order subdifferentials of f at $x \geq 0$, where $f^+(x^*) = -(-f^*)(-x^*)$ for the Young conjugate f^* of f. Let for $x, y \in X$ $f^{\vee}(x; y) \equiv \lim_{\tau \searrow 0} \frac{1}{\tau} (f(x \vee \tau y) - f(x \vee 0))$ be $\vee-$ directional derivative. Analogously let

$$f^{\wedge}(x; y) \equiv \lim_{\tau \searrow 0} \frac{1}{\tau} (f(x \wedge \tau y) - f(x \wedge 0)) \tag{1}$$

be $\wedge-$ directional derivative of f at x in the direction y. These derivatives are well-defined if moreover f is *lsc* with $x \vee 0 \in dom(f)$ and *usc* with $x \wedge 0 \in dom(f)$, respectively.

2 Main Results

Let $Conv(X)$ and $Conc(X)$ be the sets of all proper, convex and, respectively, concave functionals. Let $x \geq 0$. If $f \in Conc(X)$ and is monotone then

$$\partial^{\wedge} f(x) = (\partial_+ f(0) - (x^{\perp})_+)_+ \tag{2}$$

where $\partial_+ f(0)$ stands for positive part of the classical subdifferential of f at 0. If additionally $dom(f^+)$ is bounded and f^+ is w^*-continous on X^* then

$$f^{\wedge}(x; y) = \min\{< y, u^* >: u^* \in \partial^{\wedge} f(x)\}. \tag{3}$$

Similar formulae were obtained for $\vee-$subdifferentials. Namely, under suitable assumptions, $\partial^{\vee} f(x) = (\partial_+ f(x) - \partial_+ f(x))_+ \cap x^{\perp}$ and $f^{\vee}(x; y) = \max\{< y, u^* >: u^* \in \partial^{\vee} f(x)\}$. Moreover, if $f \in Conv(X)$ and is *lsc* and continuous at x then 1° $\partial^{\vee} f(x) = \partial_+ f(x; \circ)$ and 2° for such functions f_1, f_2 there holds

$$\partial^{\vee}(f_1 + f_2)(x) = \partial^{\vee} f_1(x) + \partial^{\vee} f_2(x) \tag{4}$$

whenever $x \in dom(f_1 + f_2)$ is a point of continuity of these functions.

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Regularity of the value function for state constrained optimal control problems

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Abstract

We provide sufficient conditions for the local Lipschitz continuity of the value function associated to the Mayer optimal problem under pure state constraints. For this aim we propose a new inward pointing condition that allows to obtain Neighboring Feasible Trajectory type results in the case of state constraint having nonsmooth boundary.

Keywords: Optimal Control, Differential Inclusions, State Constraints, Value Function.

1 Introduction

The value function is recognized to be an important tool of optimal control theory. When it is sufficiently smooth it is a solution of an associated Hamilton-Jacobi equation which can be used to express optimal synthesis and also to obtain sufficient optimality conditions. For optimal problems under state constraints the value function is, in general, discontinuous even when all data are smooth. However, if the cost function is locally Lipschitz, then exactly as for unconstrained optimal control problems, local Lipschitz continuity of the value function can be proved whenever feasible trajectories of the related differential inclusion depend on initial conditions in a Lipschitz way. In general, feasible trajectories do not enjoy this property and, in addition to standard assumptions on the dynamics (i.e. measurable in time, Lipschitz in the state variable), an inward pointing condition has to be imposed to deduce Lipschitz like behavior of feasible solutions with respect to initial conditions. Such regularity is a consequence of a Neighboring Feasible Trajectory (NFT) theorem. NFT theorems have been extensively studied and have various applications in Optimal Control Theory and Differential Games under state constraints (see for instance [2] - [3]). However, some recent counterexamples (see [1]) indicate surprisingly that conclusions of NFT theorems might not hold when the state constraint is an intersection of two half spaces in $\mathbb{R}^n$. We provide sufficient conditions, involving a new inward pointing condition, for the validity of NFT results in the case of state constraint having nonsmooth boundary.

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Topology optimization of quasistatic contact problems

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Abstract

The aim of topology optimization is to find optimal distribution of the body material within the geometrical domain resulting in its optimal shape in the sense of some shape functional. Classical approach to topology optimization problems consists in using relaxed formulations and the homogenization method. In recent years the topological derivative method [4] has emerged as an attractive alternative to analyze and solve numerically topology optimization problems. The modern mathematical background for evaluation of topological derivatives by the asymptotic analysis techniques of boundary value problems is established in [3].

In literature most papers are devoted to asymptotic and topology sensitivity analysis for elliptic boundary value problems. A few papers only, among others, [1] address this issue for the shape functionals depending on a solution to time - dependent boundary value problems. One of the reasons is that the approaches useful for stationary boundary value problems fail for evolution problems [1]. Frictional contact phenomenon between deformable bodies occurs frequently in industry or everyday life. The mathematical or engineering literature concerning this topic is rather extensive. Asymptotic and topology sensitivity analysis of solutions to unilateral stationary boundary value problems in elasticity is performed in [2].

This paper deals with the formulation of a necessary optimality condition for a topology optimization problem for an elastic contact problem with Tresca friction. Unlike in literature where the stationary contact models are used in the paper the quasistatic contact model is considered. The structural optimization problem consists in finding such topology of the domain occupied by the body in unilateral quasistatic contact that the normal contact stress along the contact boundary of the body is minimized. The volume of the body is assumed to be bounded. Using the material derivative and the asymptotic expansion methods as well as the results concerning the differentiability of solutions to quasistatic variational inequalities the topological derivative of the shape functional is calculated and a necessary optimality condition is formulated.

Keywords: quasistatic contact problem, elasticity, Tresca friction, topology optimization.

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On economic equilibrium problem involving approximate convex functions

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Abstract

The economic equilibrium problem in a reflexive Banach space setting is considered from the viewpoint of the existence issues. Convex, ε-convex and approximate convex dis-utility functions are taken into account. The theory of pseudo-monotone multivalued mappings combined with some fixed point techniques and the Galerkin method are used as the main mathematical tools.

Optimal control problems in nonsmooth contact mechanics

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Abstract

We study the optimal control problems for systems described by nonlinear evolution inclusions which are motivated by several contact problems of solid mechanics. We provide results on the existence of weak solutions to the control problems. We present some examples of the dynamic contact problem with nonmonotone nonsmooth subdifferential boundary conditions which can be formulated as the hemivariational inequalities and for which the abstract results on optimal control can be applicable.

Keywords: Hemivariational inequality, Clarke subdifferential, Optimal solution, Weak solution, Cost functional, Distributed parameter system.

1 Introduction

Several contact problems of solid mechanics which model nonmonotone and nonsmooth subdifferential boundary conditions in the contact between a viscoelastic body and a rigid foundation can be described by the dynamic hemivariational inequalities and lead to evolution inclusions.

We study the mechanical problems modeled in general by a nonlinear evolution inclusion of the form

$$y''(t) + A(t)y'(t) + By(t) + M^*\partial J(t, My(t), My'(t)) \ni f(t) \quad \text{a.e. } t \in (0, T),$$

where $A(t)$ and B are linear continuous operators from a reflexive Banach space V into its dual V^*, M is a linear continuous operator, M^* is its adjoint, ∂J denotes the Clarke subdifferential of a locally Lipschitz time-dependent function and f is prescribed.

2 Statements of problems

We deal with the following optimal control problem

$$\inf\{\, F(y, u) : y \in \mathcal{S}(u) \,\}$$

where

$$F(y, u) = l(y(T), y'(T)) + \int_0^T L(t, y(t), y'(t), u(t))\, dt$$

is the cost functional of Boltza type and $\mathcal{S}(u)$ is the solution set to the problem

$$\begin{cases} y''(t) + A(t)y'(t) + By(t) + M^*\partial J(t, My(t), My'(t)) \ni f(t) + C(t)u(t) \quad \text{a.e. } t \\ y(0) = y_0, \quad y'(0) = y_1 \\ u(t) \in U(t) \quad \text{a.e. } t \in (0, T) \end{cases}$$

and u represents a control variable, $C(t)$ is a given family of linear operators. We apply the direct method of the calculus of variations to show the existence of optimal solution.

To this end we prove that the solution set $\mathcal{S}(u)$ is not empty and the graph of the solution map $u \mapsto \mathcal{S}(u)$ is closed in suitable topologies.

It should be pointed out that, in general, optimal control problems for hemivariational inequalities are formulated as double minimization, maximization or minimax problems since usually the hemivariational inequality does not possess a unique solution.

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High-quality real-time simulation of a turbulent flame

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Abstract

We consider a 2-dimensional model of a turbulent, methane flame. The model is based on the compressible Navier-Stokes equations extended with a temperature equation and chemical reaction properties. In order to achieve a high-quality real-time simulation, a fully adaptive finite element method is used to solve the considered system of equations. The method performs adaptive mesh refinement and local adjustment of additional approximation parameters. The structure and composition of the flame, along with the numerical properties of the method, such as the mesh density, are studied. The results are compared to results obtained with a direct numerical simulation.

Keywords: Combustion, Flame, Adaptive Finite Element Method

1 Introduction and results

Flame behavior has been extensively studied e.g. for the purpose of jet engine design. Flame and smoke propagation models are used in fire-protection systems. Simplified flame models are also of interest of the entertainment industry. In this paper, we present a flame simulation technique that can be used in either case. We use a 5-step reduced mechanism of methane-oxygen combustion, which gives already reliable results while remaining relatively simple. The dynamics of the system is modelled with a variation of Navier-Stokes equations, presented e.g. in [1].

The model equations are solved with an adaptive finite element method. It is common to use adaptive mesh refinement for flame simulation. The refinement is usually performed to obtain high-resolution flame fronts. We propose an algorithm which additionally performs coarsening of the mesh at the regions of fast flow in order to obtain stability of the algorithm without decreasing the time step length. Further improvement of the convergence rate of the method is achieved by adjustment of additional approximation parameters.

We have tested the algorithm on three test cases corresponding to different equivalence ratios of the methane-oxygen mixture: $\phi = 0.6$ (a lean mixture), $\phi = 1$ (a stoichiometric mixture) and $\phi = 1.5$ (a rich mixture). The method accuracy has been verified by comparing the results to the those obtained with the direct numerical simulation. The structure and composition of the flame, along with the numerical properties of the method, such as the mesh density, have been studied.

Numerical experiments confirmed that the simulation speed, expressed in frames per second, is highly correlated with the number of mesh points, as well as the approximation error estimate. In most cases, we achieved satisfactory simulation speed, that is above the critical 16 frames per second. Better accuracy was obtained for higher equivalence ratios. The soot release was highest in the rich-mixture case. Similar behavior is observed for carbon monoxide. Rich-mixture case is related to lower oxygen supply, which results in lower reaction rates of all the reactions that consume oxygen.

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Regularization techniques for some class of hemivariational inequalities

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Abstract

We combine the regularization technique with the finite element method to approximate hemivariational inequalities with a superpotential expressed by a maximum or minimum function. These functions describe practice-oriented cases as many problems arising in mechanics and engineering show. Moreover, the regularization function for such class of functions can be explicitly computed, following [1].

Keywords: nonsmooth, HV inequalities, regularization, FEM, convergence.

1 Introduction

The hemivariational inequalities have been first introduced by P. D. Panagiotopoulos in [2] as a generalization of the variational inequalities. They involve nonconvex, generally nonsmooth energy potentials called nonconvex superpotentials. This kind of energy potentials appear if nonmonotone, generally multivalued stress-strain or reaction-displacement law are considered. For details we refer to [2, 3] and the references therein.
In this paper we first use a regularization procedure to smooth the nonsmooth superpotential. Then, the finite element approach for the regularized problem is analysed and convergence results are given. As an application we consider a bilateral contact problem with a nonmonotone friction and present some numerical results.

2 A Main Result

We denote $V = H^1(\Omega; \mathbb{R}^m)$ and define the functional $J : V \to \mathbb{R}$ by $J(v) = \int_\Gamma f(\gamma v(s))\, ds$. Here γ stands for the trace operator from $H^1(\Omega; \mathbb{R}^m)$ into $L^2(\Gamma; \mathbb{R}^m)$. For a given $g \in V^*$ we consider the problem (P): find $u \in K$ such that

$$\langle Au - g, v - u\rangle + J^0(u; v - u) \geq 0 \quad \forall v \in K, \tag{1}$$

where $J^0(u; v)$ stands for the generalized directional derivative of J and $K \subseteq V$ is a nonempty closed convex set. Using a smoothing approximation $S(x, \varepsilon)$ for $f(x)$ we approximate the non-differentiable functional J by $J_\varepsilon : V \to \mathbb{R}$ defined by $J_\varepsilon(v) = \int_\Gamma S(\gamma v(s), \varepsilon)\, ds$. The regularized problem (P_ε) now reads: find $u_\varepsilon \in K$ such that

$$\langle Au_\varepsilon - g, v - u_\varepsilon\rangle + \langle \nabla J_\varepsilon(u), v - u\rangle \geq 0 \quad \forall v \in K. \tag{2}$$

We show that the sequence $\{u_\varepsilon\}$ of solutions of the problem (2) converges strongly in V to the solution u of the problem (1).

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Nonsmooth integral functionals and its applications

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Abstract

The talk deals with the integral representation of Clarke's generalized gradient of locally Lipschitz integral functionals defined on Orlicz spaces and its applications. We present some results concerning the existence of weak solutions in Orlicz and Orlicz–Sobolev spaces.

Keywords: generalized gradient, nonsmooth locally Lipschitz functional, nonlinear viscoelasticity, partial differential inclusion, nonconvex energy, Orlicz space, Orlicz–Sobolev space.

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Infinite horizon optimal control problems - a Hilbert space approach

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Abstract

Still at the beginning of the previous century the optimal control problems with infinite horizon became very important with regards to applications in economics, where an infinite horizon seems to be a very natural phenomenon.
In general, the validity of Pontryagins Maximum Principle as necessary optimality condition was not shown up to now. In a famous counter-example of Halkin it was demonstrated that a natural extension of the transversality condition $\lim_{t\to\infty} y(t) = 0$ does not hold. This makes the numerical computation of the problem very difficult.
In this paper we use weighted Sobolev-spaces as state spaces and obtain Pontryagins Maximum principle as separation theorem in Hilbert spaces. On this way we get also adequate transversality conditions.
Applications from economics and biology are given to check if the assumptions of the theorem make sense.

Keywords: Optimal Control, Infinite Horizon, Weighted Sobolov Spaces, Maximum Principle.

Combined reformulation of bilevel programming problems

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Abstract

J. J. Ye and D. L. Zhu proposed in [1] a new reformulation of a bilevel programming problem. This reformulation compounds the value function and KKT approaches and therefore it contains not only the optimal value function restriction but also KKT constraints. In [1] partial calmness condition was also adapted to this new reformulation and optimality conditions using partial calmness were introduced.
In the talk I investigate above all two questions: what kind of advantages and difficulties compared with other reformulations exist while using combined reformulation and partial calmness condition and how could other constraint qualifications and optimality conditions be defined without using partial calmness (which is a rather restrictive, at least not generic assumption).
Since the optimal value function is in general nondifferentiable even if the lower level problem is differentiable and KKT constraints have MPEC-structure, the combined reformulation is a nondifferentiable MPEC. This MPEC-structure allowes to adapt some constraint qualifications and necessary optimality conditions from MPEC theory using disjunctive form of the combined reformulation. Compared with results from Ye and Zhu it turned out, that the necessary optimality conditions arise using MPEC-structure are exactly the same. Morover an example shows, that some of the proposed constraint qualifications based on MPEC-structure can be fulfilled.

Keywords: bilevel programming, value function reformulation, KKT reformulation, constraint qualifications, optimality conditions.

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P-factor method for nonlinear optimization

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Abstract

We present the main concept and results of the p-regularity theory (also known as p-factor analysis of nonlinear mappings) applied to nonlinear optimization problems. This approach is based on the construction of p-factor operator. The main result of this theory gives a detailed description of the structure of the zero set of an irregular nonlinear mappings. Applications include a new numerical method for solving nonlinear optimization problems and p-order necessary and sufficient optimality conditions. We substantiate the rate of convergence of p-factor method.

Keywords: nonlinear optimization, p-factor operator, p-regularity condition, kernel, singularity.

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Optimal control subject to singularly perturbed convection-diffusion-equations

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Abstract

We consider the optimal control problem

$$\min_{y,u} J(y,u) := \min_{y,u} \left(\frac{1}{2}\|y - y_0\|_0^2 + \frac{\lambda}{2}\|u\|_0^2 \right)$$

subject to a convection dominated differential state equation

$$Ly := -\varepsilon y'' + ay' + by = f + u \text{ in } (0,1),$$
$$y(0) = y(1) = 0.$$

The solutions of such singularly perturbed differential equations typically exhibit boundary layers. The optimality condition leads to the enhanced system of the state equation and its adjoint form. The change of sign of the convection term in the adjoint equation induces a boundary layer of the adjoint state at the opposite side of the domain compared to the primal state. By the coupling of the two differential equations via the optimality system those layers lead to additional boundary layers of a weaker form in the other part of the solution. Our analysis shows the layer structure of the solution of such an optimality system.

Using linear finite elements on adapted grids of Shishkin type we treat the effects of the arising layers at the boundaries of the domain. Furthermore we proof uniform error estimates with respect to the perturbation parameter ε. We show that the weak boundary layers also have an impact on the quality of numerical algorithms for solving the optimality system. Numerical results supporting our analysis are presented.

Moreover, we discuss extensions to the case of box constraints for the control u.

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The T-stationarity concept in nonsmooth optimization

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Abstract

The common motivation behind the design of stationarity concepts for minimization problems is to produce a possibly small set of candidates for locally minimal points. Whereas this often makes sense from a theoretical as well as practical point of view, there are a number of situations in which stationarity has to be seen from a different point of view. These include, for example, the design of homotopy methods as well as the investigation of optimization problems from a structural point of view.

In fact, we call a stationarity concept for a given class of optimization problems *topologically relevant*, if it allows to establish a Morse theory, that is, a nondegeneracy concept, a Morse index, an equivariant Morse lemma, a deformation theorem, and a cell attachment theorem. For example, in finite-dimensional smooth constrained optimization, Karush-Kuhn-Tucker stationarity is topologically relevant, and for mathematical programs with complementarity constraints (MPCCs), C-stationarity turned out to be topologically relevant.

Recent results for mathematical programs with vanishing constraints (MPVCs) illustrated that the topologically relevant stationarity concept is not necessarily one which is already known. This motivated the introduction of T-stationarity for MPVCs, where T stands for *topologically relevant.*

After explaining T-stationarity for different classes of smooth and nonsmooth optimization problems, we point out implications concerning numerical methods for MPCCs and MPVCs as well as connections to the design of Newton methods for nonsmooth optimization problems.

This is joint work with Dominik Dorsch and Vladimir Shikhman, RWTH Aachen University, Germany.

Keywords: Morse theory, homotopy method, C-stationarity, MPEC, MPVC, smoothing method, nonsmooth Newton method.

Higher-order conditions for strict local Pareto minima in terms of generalized lower and upper directional derivatives

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Let X, Y be normed spaces. For a function $f : X \to \mathbb{R}$, we consider the following m-th order lower and upper directional derivatives (Studniarski, 1986)

$$\underline{d}^m f(\bar{x}; y) := \liminf_{(t,v)\to(0^+,y)} \frac{f(\bar{x}+tv)-f(\bar{x})}{t^m},$$
$$\overline{d}^m f(\bar{x}; y) := \limsup_{(t,v)\to(0^+,y)} \frac{f(\bar{x}+tv)-f(\bar{x})}{t^m}.$$

For a vector-valued function $f = (f_1, ..., f_p) : X \to \mathbb{R}^p$, let

$$\underline{d}^m f(\bar{x}; y) = (\underline{d}^m f_1(\bar{x}; y), ..., \underline{d}^m f_p(\bar{x}; y)),$$
$$\overline{d}^m f(\bar{x}; y) = (\overline{d}^m f_1(\bar{x}; y), ..., \overline{d}^m f_p(\bar{x}; y)),$$

where each component of these vectors belongs to $\mathbb{R} \cup \{-\infty, \infty\}$.

We shall deal with the following multiobjective optimization problem:

$$\min\{f(x) : x \in S\}, \tag{1}$$

where

$$S := \{x \in X : -g(x) \in D, x \in C\},$$

$f : X \to \mathbb{R}^p$ and $g : X \to Y$. We assume that C and D are nonempty closed subsets of X and Y, respectively, and D is a convex cone, $D \neq Y$. The minimization in (1) is understood with respect to the partial order defined by the positive cone $\mathbb{R}^p_+ = [0, \infty)^p$.

Definition 1 (Jiménez, 2002) *Let m be a positive integer, and let $\bar{x} \in S$. We say that $\bar{x}$ is a* strict local Pareto minimizer of order m *for* (1)*, denoted $\bar{x} \in \mathrm{StrL}(m, f, S)$, if there exist $\alpha > 0$ and a neighborhood U of $\bar{x}$ such that $(f(x) + \mathbb{R}^p_+) \cap B(f(\bar{x}), \alpha \|x - \bar{x}\|^m) = \emptyset$ for all $x \in S \cap U \backslash \{\bar{x}\}$, where $B(z, \delta) := \{u \in \mathbb{R}^p : \|u - z\| < \delta\}$.*

We denote by $K(S, \bar{x})$ the contingent cone to S at $\bar{x}$.

Theorem 1 *Let $\bar{x} \in \mathrm{StrL}(m, f, S)$. Suppose that $\operatorname{int} D \neq \emptyset$ and*

$$dg(\bar{x}; y) := \lim_{(t,v)\to(0^+,y)} \frac{g(\bar{x}+tv)-g(\bar{x})}{t}$$

exists for all $y \in X$. Then there exists $\beta > 0$ such that $\overline{d}^m f(\bar{x}; y) \notin B(0, \beta \|y\|^m) - \mathbb{R}^p_+$ for all $y \in K(C, \bar{x}) \cap \{u \in X : dg(\bar{x}; u) \in -\operatorname{int} D\}$.

Theorem 2 *Let $\dim X < +\infty$, and let $\bar{x}$ be a feasible point for problem* (1)*. Suppose that $dg(\bar{x}; y)$ exists for all $y \in X$. If $\underline{d}^m f(\bar{x}; y) \notin -\mathbb{R}^p_+$ for all $y \in K(C, \bar{x}) \cap \{u \in X : dg(\bar{x}; u) \in -\mathrm{cl\ cone}(D + g(\bar{x}))\}$, then $\bar{x} \in \mathrm{StrL}(m, f, S)$.*

Some applications of optimal control in sustainable fishing in the Baltic Sea

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Abstract

Issues related to the implementation of dynamic programming for optimal control of a three-dimensional dynamic model (the fish populations man- agement problem) are presented. They belong to a class of models called Lotka-Volterra models.

The problem of optimal harvest policy is solved for the control of various classes of its behaviour. The optimal strategy for catching a 3-population system cod-herring-sprat for a time interval of 20 years was calculated using standard software.

The second focus will be the Discrete Dynamic Programming and its applying in ecological management problems.

Numerical implementation of MSE

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Abstract

The methods of numerical solution of optimal control problems are generally divided into direct and indirect. The indirect approach typically requires an a priori knowledge of the structure of the optimal control. The alternative, direct approach [1] requires parameterizing controls and discretizing differential equations to form an NLP problem which may be solved using standard optimization methods. Specialized software tools for optimal control include Fortran codes [2] and MATLAB programs. The MSE package [3] presented here belongs to the latter class of direct methods. Basic concepts of the MSE method are discussed in the accompanying contribution [4].

Our aim was to create a tool, able to leverage minimal (in the sense of parameter number), consistent control representations, and systematic control structure evolution by using the MSE. The package covers a large class of optimal control problems for ODE including: fixed and free horizon problems, control and state constrained problems, problems with state-feedback and non state-feedback singular arcs, single and multi-interval formulations and switched systems. The tool is still under development. Its main implemented features include: fixed and variable-step solvers with full support of discontinuities, BFGS based or Newton type (with curvilinear search) optimizers, polynomial interpolation schemes, variable initial values for structural arcs (as decision variables). The MSE package is equipped with a GUI for problem/options/test run settings definition, test run execution and result analysis. Iteration history and structure evolution plots can be generated. A limited support for automatic differentiation is offered. We expect that more features will be implemented soon, among them: multi-interval problem formulations, event support in integrators and C code generation for problem functions, integrators and optimizers. In a longer perspective we consider adding an outer loop for penalized problems (automatic penalty coefficients selection), and support for DAE and DDE models.

The current capabilities of the MSE package are illustrated with some examples including a simple bang-bang problem, a single input semi-batch fermentation process with bang-singular control, and a problem requiring prototype adjoints. We wish to emphasize the importance of consistent procedures [3] implemented in the MSE package.

Keywords: Optimal control, Dynamic optimization, Numerical methods.

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A priori error analysis for space-time finite element discretizations of parabolic optimal control problems

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Abstract

In this talk we discuss our approach to discretization of optimization problems governed by parabolic partial differential equations. We use Galerkin finite element methods for both temporal and spatial discretizations. This allows for the equivalence of the optimize-then-discretize and the discretize-then-optimize approaches. Moreover we present systematic a priori error analysis for problems with different kind of inequality constrains.

On goal-oriented error estimation for elliptic optimal control problems

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Abstract

Goal-oriented error estimation for PDE problems aims at estimating the error in a certain quantity of interest instead of some norm, and controls the mesh refinement process such as to create meshes adapted to this purpose. In optimal control problems, the natural quantity of interest is the objective. This paper highlights the difference between two seemingly similar formulations of the objective and discusses the impact on accuracy matching for inexact Newton. Hierarchical error estimators are proposed for computing the weight functions. Issues of global error transport in boundary control problems are considered. The concept is illustrated with numerical examples

Keywords: goal-oriented error estimation, adaptive mesh refinement, optimal control, Newton's method

1 Introduction

For a convex linear-quadratic optimal control problem

$$\min_{y,u} J(y,u) \quad \text{s.t.} \quad Ay + Bu = b_\lambda$$

with solution y_*, u_*, the most natural quantity of interest is the objective J itself. Accordingly, Becker, Kapp and Rannacher have proposed to consider $E_h = J(y_h, u_h) - J(y_*, u_*)$ as the quantity of interest for estimating the error of an approximate solution y_h, u_h [1]. However, if optimization is performed in order to apply the computed control u_h in a physical system, the discretized state y_h plays no role at all. Instead, what determines the practical performance is the black-box error $\tilde{E}_h = J(y(u_h), u_h) - J(y_*, u_*)$. Even though the difference looks negligible at a first glance, it turns out to be decisive in several situations.

2 Main Results

Goal oriented error estimation w.r.t. the black-box error $\tilde{E}_h$ is somewhat more complex than w.r.t. the all-at-once error E_h, but can give dramatically different results. In particular when corner singularities prevent a small state error $y_h - y_*$, the control error $u_h - u_*$ may nevertheless be rather small, which is relevant for finite-dimensional and boundary control. A structural advantage of $\tilde{E}_h$ is that it induces a norm on the control error.
Hierarchical error estimators can straightforwardly be used to compute the residual weight functions. It turns out, however, that care has to be taken when localizing the error estimator in order not to miss global error transport, which is crucial for boundary control. The black-box error estimation concept can be transfered to the accuracy maching of Newton's method for nonlinear problems. This results in a new criterion to determine the accuracies of normal and tangential steps, respectively.

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Automatic differentiation in the optimization of imaging optical systems

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Abstract

One step in the design of imaging optical systems is the optimization of free system parameters in order to maximize image quality. The objective function typically requires ray tracing, i.e. the computation of the paths rays travel through the optical system while being refracted of reflected at the various surfaces. We consider this optimization as an optimal control problem with discrete time steps. We have implemented the ray tracing procedure and have successfully used automatic differentiation to obtain derivatives of the objective function. The application of automatic differentiation in lens design is novel to the best of our knowledge. We show that automatic differentiation is superior to numerical differentiation also in the optimization of optical system. Current efforts include introducing constrained optimization to enforce geometrical constraints, thus avoiding non-fabricable designs, total internal reflection and rays missing a surface. This ensures that the objective function can always be evaluated.

Penalty and Barrier Methods for optimal control problems with control constraints

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Abstract

Penalty and barrier methods turned out to be efficient techniques for the solution of a wide class of optimal control problems. We apply these algorithms to linear-quadratic control problems with distributed control and elliptic state equation. One can show linear convergence of auxiliary solutions for quadratic loss penalty. Further, we discuss control reduction and finite element discretization and deduce appropriate error bounds.

Keywords: optimal control, penalty barrier methods, control reduction, semi-smooth newton method, finite element method, order of convergence.

1 Introduction

Let $\Omega \subset \mathbb{R}^2$ be a bounded domain and $y_d, b \in L_\infty(\Omega)$, $\alpha > 0$ be given. We consider

$$J(u) := \frac{1}{2}\,\|S\,u - y_d\|^2_{L_2(\Omega)} + \frac{1}{2}\,\|u\|^2_{L_2(\Omega)} \to \min! \quad \text{s.t.} \quad u \in U_{ad}. \tag{1}$$

Thereby, $S : U := L_2(\Omega) \to V := H^1(\Omega) \hookrightarrow U$ is the solution operator of an elliptic PDE. The set of admissible controls is defined by $U_{ad} := \{u \in U\colon u \leq b\}$. We treat the constraint $u \in U_{ad}$ by a penalty method and define a sequence of unconstrained problems

$$J(u,s) := J(u) + \int_\Omega \Phi(u - b, s)\,\mathrm{d}\Omega \to \min! \quad \text{s.t.} \quad u \in U \tag{2}$$

with penalty parameter $s > 0$ and solutions $u(s)$. Thereby, $\Phi : \mathbb{R}\times\mathbb{R}_+ \to \mathbb{R}$ is an arbitrary penalty or barrier function (e.g. logarithmic barrier, quadratic loss penalty, SUMT, ...).

2 Main Results

We show existence of a unique solution of (2) and deduce an optimality system (cf. [1]). Further, we prove strong convergence of auxiliary solutions and rate of convergence 1, i.e.

$$\|u(s) - \bar{u}\| \leq C\,s \qquad \forall\, s > 0$$

holds with a constant $C > 0$ (cf. [2]). One can replace the control by a pointwise resolution of the necessary optimality condition, namely $u(p; s)$. Using this, the optimality system is equivalent to an operator equation, which depends on the state and adjoint state only. This equation can be solved by a semi-smooth Newton method which converges locally quadratically (cf. [3]). Last, we consider finite element discretization with linear C^0-elements and assess that the discretization error is $\mathcal{O}(h^2)$.

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Optimal control problem of a rotating Timoshenko beam and its numerical solution

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Abstract

We continue our work concerning controllability problem for Timoshenko beams rotating in a horizontal plane. The optimal control problem is stated and a numerical method of obtaining the solution is presented.

Keywords: Rotating beams, controllability, rest-to-rest control, optimal control.

This work summarizes our results concerning controllability of a Timoshenko beam clamped to a motor disk slowly rotating in a horizontal plane (see references). We consider a linearized model in dimension-free formulation from [2], namely

$$\begin{aligned} &\ddot{w}(x,t) - w''(x,t) - \xi'(x,t) = -\ddot{\theta}(t)(r+x), \\ &\ddot{\xi}(x,t) - \gamma^2\xi''(x,t) + w'(x,t) + \xi(x,t) = \ddot{\theta}(t) \end{aligned} \tag{1}$$

with $x \in (0,1)$ and $t > 0$, where $w(x,t)$ denotes the deflection of the center line of the beam, $\xi(x,t)$ – the rotation angle of the cross section area, $\theta(t)$, $r > 0$ – the rotation angle and the radius of the motor disk, and $\gamma > 0$ – a parameter connected with the physical properties of the beam. In addition we have boundary conditions given by $w(0,t) = \xi(0,t) = w'(1,t) + \xi(1,t) = \xi'(1,t) = 0$ for $t \geq 0$. Further on we assume that $\gamma = 1$. Let time $T > 0$ (large enough) and position $\theta_T \in \mathbf{R}$ be given. We steer the beam by angular acceleration of the disk, that is by $\ddot{\theta}$, for our convenience let us denote $u(t) = \ddot{\theta}(T-t)$. We want to find a control $u \in L^2[0,T]$ such that it moves the beam from the position of rest at time $t = 0$ and angle $\theta = 0$, i.e. $w(x,0) = \dot{w}(x,0) = \xi(x,0) = \dot{\xi}(x,0) = \theta(0) = \dot{\theta}(0) = 0$ for $x \in [0,1]$, to the position of rest at time $t = T$ and angle θ_T, i.e. $w(x,T) = \dot{w}(x,T) = \xi(x,T) = \dot{\xi}(x,T) = \dot{\theta}(T) = 0, \theta(T) = \theta_T$, $x \in [0,1]$. Moreover, we want u to be optimal in the following sense: $\int_0^T u^2(t)dt \to \min$. After using some spectral properties of the differential operator connected with our equation (1) we show that the optimal solution of the obtained moment problem can be approximated by optimal solution of another non-Fourier moment problem. In the end the latter can be reduced to solving a finite number of linear equations which in turn can be solved numerically.

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Subdifferential determination of some lsc functions in Banach space

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Abstract

During the talk a new large class of extended-real-valued functions which are subdifferentially determined is to be presented. Functions from the class are called essentially directionally smooth functions. It will be shown that lower semicontinuous extended-real-valued convex functions (more generally the approximate convex functions) and locally Lipschitz continuous arc-wise essentially smooth functions are included in the class. Moreover, if we take the sum of two functions from those two above classes, i.e., the convex and locally Lipschitz continuous arc-wise essentially smooth ones, we are still in the class, although the sum of a lower semicontinuous extended-real-valued convex function with a locally Lipschitz continuous function can be neither convex nor locally Lipschitz continuous. In fact, the essential directional smoothness property is stable under addition.

Generalized hybrid steepest descent methods for some variational inequality problem

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Abstract

We consider the following variational inequality problem $VIP(\mathcal{F}, C)$ in a real Hilbert space $\mathcal{H}$: find $\bar{u} \in C$ such that $\langle \mathcal{F}\bar{u}, z - \bar{u}\rangle \geq 0$, for all $z \in C$, where $\mathcal{F} \colon \mathcal{H} \to \mathcal{H}$ is Lipschitz continuous and strongly monotone, $C := \bigcap_{i \in I} \operatorname{Fix} U_i \neq \emptyset$ and U_i have the following property: $\langle u - U_i u, z - U_i u\rangle \leq 0$ for all $u \in \mathcal{H}$ and $z \in \operatorname{Fix} U_i$, $i \in I := \{1, 2, ..., m\}$. In particular metric projections P_{C_i} onto closed convex subsets $C_i \subseteq \mathcal{H}$ and subgradient projections P_{f_i} for continuous convex function $f_i : \mathcal{H} \to \mathbb{R}$ with $S(f_i, 0) := \{u \in \mathcal{H} : f_i(u) \leq 0\} \neq \emptyset$ have this property. It is well known that $VIP(\mathcal{F}, C)$ has a unique solution. We propose the following method for $VIP(\mathcal{F}, C)$: $u^{k+1} = T_k u^k - \lambda_k \mathcal{F} T_k u^k$, where $T_k := (\,Id) + \alpha_k (U_{i_k} - \mathrm{Id})$, $\alpha_k \in [0, 2]$ and $\{i_k\} \subseteq I$ is a control sequence. If we take $T_k := T$ for all $k \geq 0$, then we obtain a hybrid steepest descent method (see [1]). We give sufficient conditions for the convergence of sequences generated by the proposed method to the solution of $VIP(\mathcal{F}, C)$. The results show an application of a convergence theorem presented at the same conference by A. Cegielski.

Keywords: variational inequality problem, hybrid steepest descent method, metric projection, subgradient projection.

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